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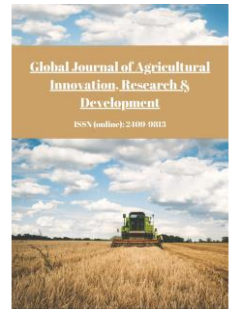

Program Control of Root Crops

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ABSTRACT

The theoretical foundations of programmatic control of root crops, implemented on a daily time scale during a specific period of vegetation, have been developed. The programmatic control level is crucial, as it is through it that the set control goal is achieved for the adopted crop cultivation conditions. The sequence of technological actions generated by this control level is called a control program, or simply a program. Therefore, this type of control is programmatic. If crop cultivation conditions remained unchanged, the control programs obtained in advance would remain unchanged in real time. Therefore, changing conditions require adjustments to the control programs, which is a real-time task. The sequence of technological operations at this control level includes the application of mineral fertilizers and irrigation at pre-selected points in time. To date, results have been obtained on programmatic control only for crops with above-ground commercial biomass (grass, grain). A theoretical basis for root crops has not yet been developed. This article presents the results of programmatic control of the condition of root crops whose commercial mass is located in the soil. They were obtained using the example of sugar beet management.

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1. Introduction

In accordance with the concept of general management of agricultural technologies in precision agriculture [1], the problem of software control of agricultural technologies is solved in two stages. This is due to the fact that the control object, which is an agricultural field, represents two independent complex dynamic systems. The first of these is the sowing of an agricultural crop, and the second is the soil environment, through which the main control of the crop condition is carried out. The control itself is a sequence of technological operations in the form of doses of mineral fertilizers and irrigation rates. Until now, the solution to the problem of software control was carried out mainly for crops with above-ground commercial mass (grains, perennial grasses) [2-11]. Management of the condition of root crops has not been considered. This is due to the fact that the commercial part of the biomass of these crops is located in the soil and is inaccessible to remote sensing tools, which are a modern effective means of monitoring the condition of crops. Based on monitoring information, it is possible to construct mathematical models of the condition of root crops and solve problems of managing their condition. As with crops with above-ground biomass, the goal of programmatic management is to achieve a target crop yield at the end of the growing season. The first stage of the overall programmatic management task involves developing a program for changing soil parameters to ensure the achievement of this goal. This does not take into account physical constraints on management, nor does it demonstrate how to implement them. Solving the problem at this stage not only significantly simplifies the overall management task but also has independent significance. First, it demonstrates the achievability of the stated management goal and the feasibility of solving it, and second, it demonstrates the potential yield of the crop.

The second stage involves developing a sequence of technological operations for nutrient application and irrigation that ensures the closest approximation of the dynamics of soil chemical parameters to the optimal program obtained in the first stage. The resulting management program is maintained throughout the growing season and generates forecasts for the dynamics of soil parameters and crop biomass, along with an estimate of the most probable yield, which will most often be below potential [1, 2].

2. Materials and Methods

The mathematical basis of the problem is a model of the dynamics of biomass parameters over the sugar beet growing season, supplemented by a model of the dynamics of soil environment parameters, including the content of essential nutrients and soil moisture content.

A model of the dynamics of sugar beet crop biomass parameters in expanded vector-matrix form over the growing season

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}d_N(t)\\ d_K(t)\\ d_P(t)\end{bmatrix}+\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ d_{31} & d_{32} & d_{33} & d_{34}\end{bmatrix}\begin{bmatrix}v_N(t)\\ v_K(t)\\ v_P(t)\\ v_4(t)\end{bmatrix}+\begin{bmatrix}c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & c_{23}\\ 0 & 0 & c_{33}\end{bmatrix}\begin{bmatrix}f_1(t)\\ f_2(t)\\ f_3(t)\end{bmatrix}+\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\\ \xi_3(t)\end{bmatrix}, \tag{1}$$

or in symbolic vector-matrix form

$$X^{\cdot}=AX(t)+Bd(t)+DV(t)+CF(t)+Z(t), t\in(t_0,T), X(0)=X_0. \tag{2}$$

where: $t\in(0,T)$ is the vegetation period, days, x_1, x_2, x_3 are the average field-area value of the total biomass of crop leaves (tops), raw above-ground biomass of crop leaves (tops), and total biomass of crop roots, cwt×ha^{-1}; d_N, d_K, d_P are the average field-area doses of foliar application of nitrogen, potassium, and phosphorus fertilizers, respectively, kg ha^{-1}; v_N, v_K, v_P is the average field-area content of the active substance, respectively, nitrogen, potassium, and phosphorus in the soil, kg ha^{-1}; v_4 is the average field-area soil moisture content, mm; f_1 is the average daily air temperature, °C, f_2 is the average daily solar radiation, W m^{-2}; f_3 is the average daily precipitation intensity, mm; ξ_1, ξ_2, ξ_3, ξ_4 – random modeling errors taking into account unobservable and unaccounted factors, which are random processes with zero means and variances σ^2_1, σ^2_2, σ^2_3, σ^2_4; a_{11}–a_{44} – parameters of the dynamic matrix of model (1),(2); b_{11}-b_{23} – parameters of the control transfer matrix of model (1), (2); d_{31}–d_{34} – parameters of the matrix of the relationship between the parameters of the state of the crop biomass and the soil parameters; c_{11}–c_{43} – parameters of the matrix of transfer of climatic disturbances of model (1), (2); A, B, D, C, K – model matrices in accordance with the expanded form of the model; Z is the vector of random modeling errors.

The state parameter model (1), (2) allows for forecasting sugar beet yields, thereby solving the management problem without resorting to numerous other yield factors that complicate the solution. Furthermore, all model variables are observable, allowing for the refinement of model parameters in real time. Model variables (1), (2) allow for the evaluation and forecasting of the management objective, making them goal-forming.

Model of parameters of the state of the soil environment (SE) in expanded vector-matrix form

$$\begin{bmatrix}\dot{v}_N\\ \dot{v}_K\\ \dot{v}_P\\ \dot{v}_4\end{bmatrix}=\begin{bmatrix}a_{11}&0&0&a_{14}\\0&a_{22}&0&a_{24}\\0&0&a_{33}&a_{34}\\0&0&0&a_{44}\end{bmatrix}\begin{bmatrix}v_N(t)\\v_K(t)\\v_P(t)\\v_4(t)\end{bmatrix}+\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}D_N(t)\\D_K(t)\\D_P(t)\\D_4(t)\end{bmatrix}+\begin{bmatrix}0&0&c_{13}\\0&0&c_{23}\\0&0&c_{33}\\c_{41}&c_{42}&1\end{bmatrix}\begin{bmatrix}f_1(t)\\f_2(t)\\f_3(t)\end{bmatrix}-\begin{bmatrix}m_{11}&0&m_{13}\\m_{21}&0&m_{23}\\m_{31}&0&m_{33}\\0&m_{42}&0\end{bmatrix}\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\end{bmatrix}, \quad (3)$$

or compact symbolic vector-matrix form

$$\dot{V} = A_V V(t) + B_V D_V(t) + C_V F(t) - M X(t) \quad (4)$$

where: D_N, D_K, D_P, D_4 are application rates of mineral fertilizers (nitrogen, potassium, phosphorus) and irrigation rates, respectively, kg×ha^{-1} for mineral fertilizers and tn×ha^{-1} for irrigation rates. Here, irrigation rates refer to water consumption per hectare of crop area.

The goal of management in this problem is to achieve a specified crop yield at the end of the growing season.

This goal is met by the following optimality criterion, which has the following form:

$$J(T) = (x_3(T) - x_3^*)^2, \quad (5)$$

where: x_3 is the specified yield value.

The result of solving the problem at the first stage is a program for changing soil environment parameters, ensuring the achievement of the stated management objective. The problem is solved with a zero foliar fertilization vector (d=0). This vector must be used for real-time management.

In accordance with the maximum principal scheme, the Hamiltonian for system (2) and optimality criterion (5) has the form [1, 6].

$$H(t) = \Psi^{T}\,(AX(t) + DV(t) + CF(t)), \quad (6)$$

where Ψ is the vector of conjugate variables, which is the solution of the system

$$\dot{\Psi} = -\frac{\partial H}{\partial X} = -A^{T}\Psi,$$
$$t \in (T,0),\ \Psi(T) = \frac{\partial J(T)}{\partial X} = 2\begin{bmatrix}0\\0\\x_3 - x_3^*\end{bmatrix}. \quad (7)$$

Given the notations introduced, the step-by-step algorithm for solving the problem at the first stage is the following sequence of operations:

Step 0. The initial conditions are set: the vector of crop condition parameters X0 and the initial value of the control program $V(t)=V_0$ (constant values of the vector components over the entire interval). The average long-term values F(0, *T*) of the climate disturbance vector is adopted. The cyclic variable *i*=0 is adopted.

Step 1. The system $X^{\cdot} = AX(t) + DV(t) + CF(t)$ is solved in direct time on the interval $t\in(0, T)$, resulting in a vector array $X_i(t)$.

Step 2. The system is solved for the adjoint variable

$$\dot{\Psi}_i = -\frac{\partial H}{\partial X_i} = -A^T\Psi_i,\ t\in(T,0),\ \Psi_i(T) = \frac{\partial J(T)}{\partial X} = 2\begin{bmatrix} 0 \\ 0 \\ x_3 - x_3^* \end{bmatrix}.$$

in reverse time on the interval, resulting in a vector array Ψi(–t), which unfolds in time Ψi(t).

Step 3. The next approximation of the program for controlling the vector of parameters of the chemical state of the soil is found

$$V_{i+1}^*(t) = V_i^*(t) - \Delta_i^* GR_i(t),$$
$$GR_i(t) = \frac{\partial H}{\partial V_i}(t) = D^T\Psi_i(t).$$

Step 4. The next approximation of the initial conditions at time *t*=0 is found

$$X_{i+1}^*(0) = X_i^*(0) - \Delta_i \Psi_i(0).$$

Step 5. The cyclic variable *i*=*i*+1 is accepted, and the transition to step 1 is performed until the condition is met.

$$J_i(T) \leq \delta.$$

As a result of solving the problem at the first stage, a program for changing the parameters of the soil condition is formed throughout the entire vegetation interval, as well as a program for the potential development of crops, which is obtained by substituting optimal programs for changing the parameters of the chemical condition of the soil into the model of crop condition parameters.

The goal of management at the second stage of the task is: "ensuring the closest approximation to the optimal program for changing the nutrient content and moisture content in the soil, obtained at the previous stage through independent selection of the amounts of fertilizing and watering." In this case, fertilizing is carried out at fixed points in time of the onset of the following phenological phases: s = 1 (4th leaf, t_1 = 12 days), s = 2 (leaf closure, t_2 = 73 days), s = 3 (technical maturity, t_3 = 120 days).

The control task at this stage is to find the sequence of the vector of fertilizer and irrigation doses $D_V(t)$ that meets the minimum criterion

$$J = \int_0^T [(V^*(t) - V(t))^T G(V^*(t) - V(t))]\, dt, \tag{8}$$

where G is the weight matrix by which the biomass structure is regulated.

The Hamiltonian of model (4) and optimality criterion (8) has the following form

$$H = [(V^*(t) - V(t))^T G(V^*(t) - V(t))] +$$
$$+\Psi^T[A_V V(t) + B_V D_V(t) + C_V F(t) - MX(t)].$$

The Hamiltonian of model (4) and optimality criterion (8) has the following form

$$H = [(V^*(t) - V(t))^T G(V^*(t) - V(t))] +$$
$$+\Psi^T[A_V V(t) + B_V D_V(t) + C_V F(t) - M X(t)].$$

The algorithm for finding this sequence includes the following steps.

Step 1. A cyclic variable i=0 is set, the initial approximation of the sequence of dose vectors $D_{01i}(t)=\{D_{01i}(t_1), D_{02i}(t_2), D_{03i}(t_3)\}$, and the average long-term values of climatic parameters F(t).

As stated above, the climate parameters are: f_1 is the average daily air temperature, °C, f_2 is the average daily solar radiation, W×m^{-2}; f_3 is the average daily precipitation intensity, mm.

Step 2. For a given interval (0, T) and the program of potential crop development X*(t) obtained in the first stage, the system is solved in direct time on the interval

$$t \in (0, T), V_i(0) = V_{i0} \ ,$$

the system is being solved

$$\dot{V} = A_V V(t) + B_V D_V(t) + C_V F(t) - M X(t).$$

as a result, we obtain a vector array Vi(t).

The criterion J_i is calculated: if J_i<δ, then stop, otherwise go to step 3.

Step 3. Solve the system for the adjoint variable

$$\dot{\Psi}(t) = -\frac{\partial H(t)}{\partial V_i} = -\frac{\partial}{\partial V_i}\{[(V^*(t) - V_i(t))^T G(V^*(t) - V_i(t))]$$
$$+ \Psi^T[A_V V(t) + B_V D_V(t) + C_V F(t) - M X(t)]\} =$$
$$= -[2G(V^*(t) - V_i(t)) + A_V^T \Psi_i(t)],$$

in reverse time on the interval

$$t \in (T, 0_1), \Psi_i(T) = 0.$$

As a result, we obtain a vector array Ψi(-t), which unfolds in direct time Ψi(t).

Step 4. The next approximation of the vector of fertilizer and irrigation doses $D_n(t)$ is found, where n=1,2,3 are the numbers of the crop phenophases

$$D_{ni+1} = D_{ni} - \Delta_i^* \frac{\partial H}{\partial D_{ni}},$$

or

$$\text{if } D_{ni} \in \Omega_D;$$
$$D_{ni+1} = D_{ni}, \text{ if } D_{ni+1} \notin \Omega_D.$$

Proceed to step 2.

3. Experimental Results

To test the problem, a software package was developed, including a mathematical model identification unit and a program control generation unit. Experimental data obtained at the experimental site of the Agrophysical Research Institute, located in the Gatchina District of the Leningrad Region, were used for testing.

The results of the testing are presented in Fig. (**1**-**4**).

Fig. (**1**) shows the process of identifying the mathematical model (1), (2). As can be seen from the graphs, the identification is stable, without interruptions. This ensures high process accuracy, with errors not exceeding 5–20%.

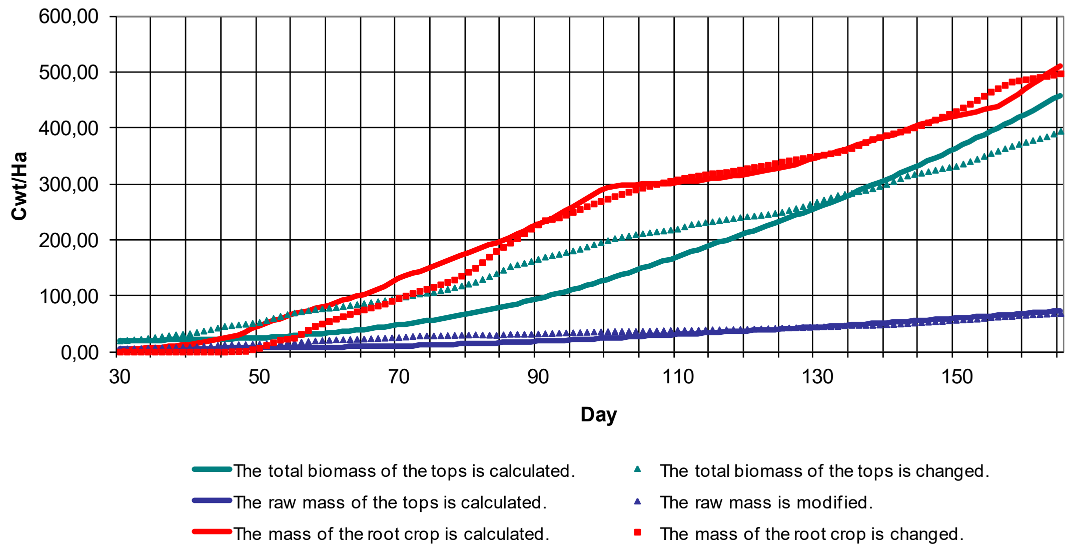

Figure 1: The process of identifying a mathematical model of sugar beet condition parameters.

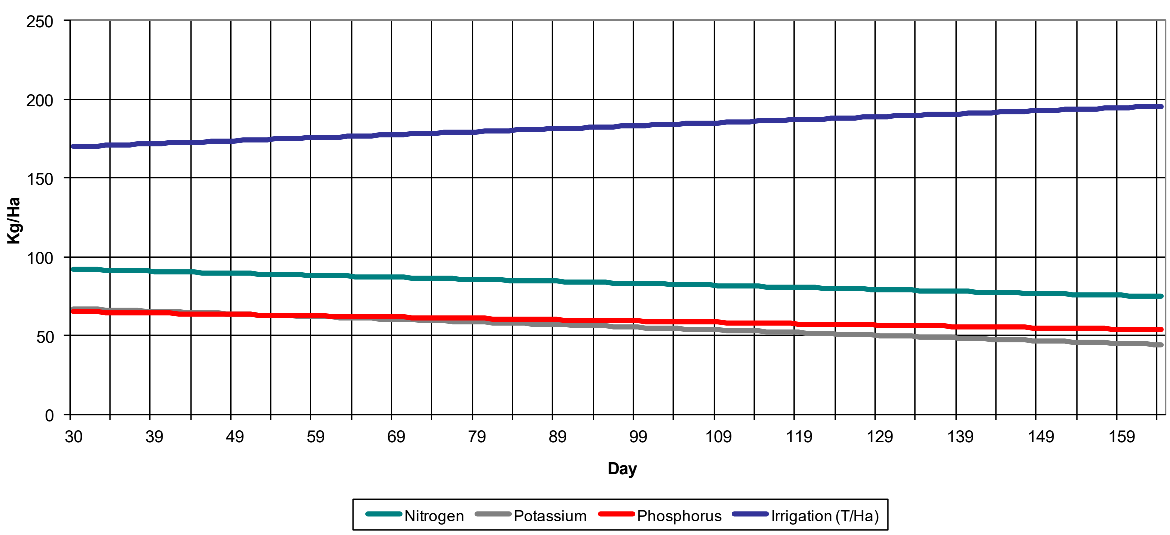

Figure 2: Optimal programs for changing (evolving) soil environment parameters.

Fig. (**2**) shows the optimal programs for changing (evolving) soil environment parameters. They are the result of solving the program control problem at the first stage and ensure the achievement of the required crop yield at the end of the growing season. This result is of purely theoretical significance and serves as a guideline for developing the control program.

Such a program is shown in Fig. (**3**). It ensures that the soil parameter modification program approaches the optimal program shown in Fig. (**2**), thereby achieving the specified control goal.

Fig. (**4**) shows the forecast for sugar beet condition parameter dynamics under the influence of the optimal control program. As can be seen from the graphs, the crop yield reaches the target value of 600 cwt×ha^{-1}.

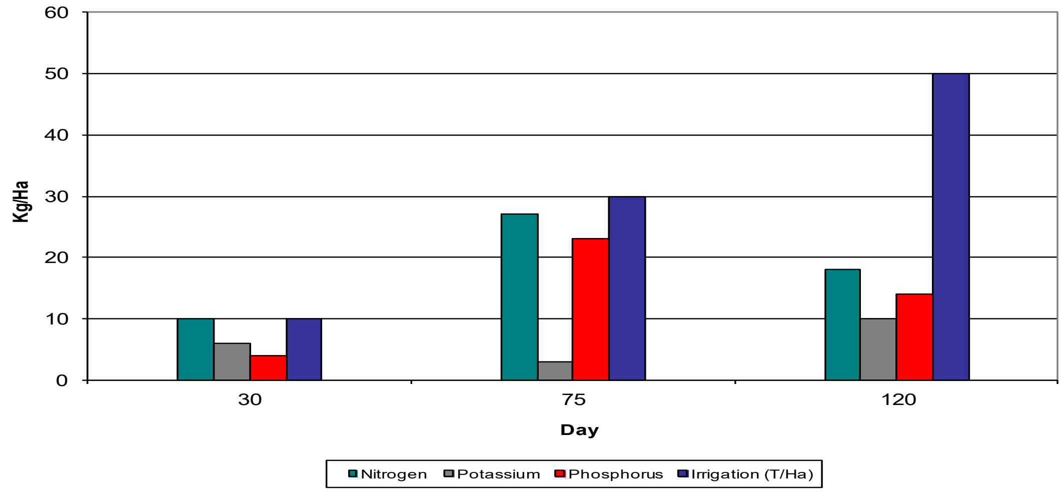

Figure 3: Sugar beet condition parameter control program.

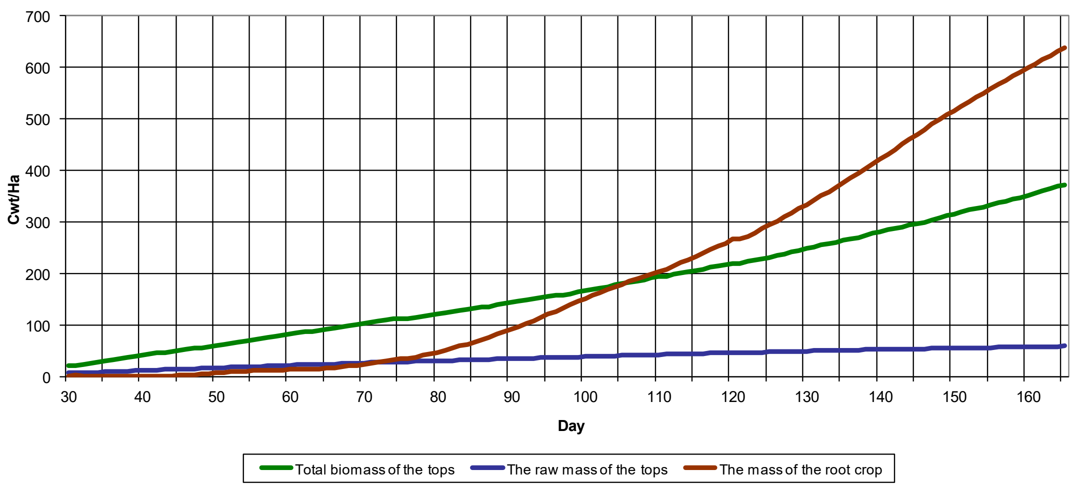

Figure 4: Forecast of sugar beet crop development under the influence of an optimal management program.

4. Discussion

According to a previously developed concept, the general problem of agricultural technology management comprises three levels of control: strategic, programmatic, and real-time [1, 4, 5]. The first two levels are planning and are solved outside of real time. These levels are currently absent from modern precision farming systems. This significantly limits modern farmers' ability to implement controlled agricultural technologies that maximize the genetic potential of crops and utilize available resources most economically. In terms of both significance and information content, the programmatic control problem is fundamental, as it implements the basic management function. This function is fully reflected in the real-time control problem, which already takes into account the actual condition of the crops.

A significant advantage of the proposed solution is that planning agricultural technologies for the next growing season is solved as an optimal control problem. This approach enables the application of methods from the general theory of dynamic systems control, thereby achieving the best possible results and utilizing the most modern theories and methods.

The novelty of the solution, compared to other similar software control problems, lies in the new mathematical models and algorithms that reflect the dynamics of the state parameters and the morphological structure of sugar beet crops.

5. Conclusions

A new theoretical framework for programmatically managing the condition of root crops, whose commercial biomass is located in the soil environment and is inaccessible to Earth remote sensing, is proposed. This problem is solved by developing new mathematical models of the plant-soil system, which reflect the interrelationships between all system elements. The overall control problem is solved in two stages. In the first stage, a program for changing soil environment parameters throughout the growing season is developed, ensuring the achievement of the target sugar beet yield. In the second stage, the optimal sequence of technological operations is determined, ensuring the maximum approximation of the predicted program for changing soil environment parameters to the program obtained in the first stage. Testing of the proposed control methodology on a prototype control system demonstrated its feasibility.

Conflict of Interest

The authors declare that there is no conflict 0f interest regarding the publication of this manuscript.

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